Logic or Logics?

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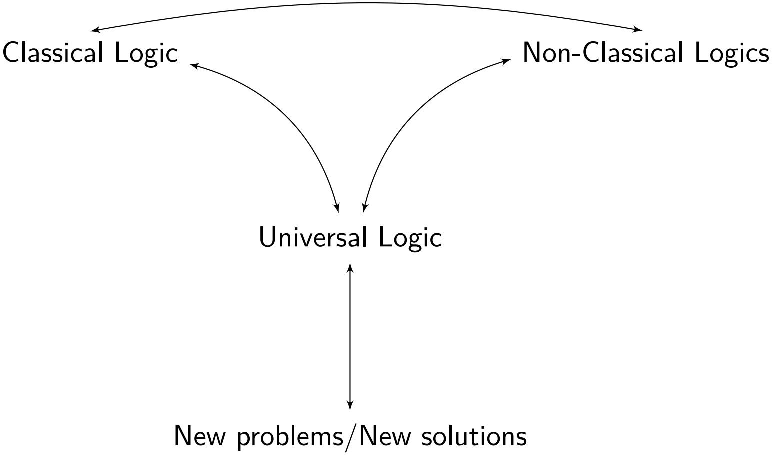
Introduction

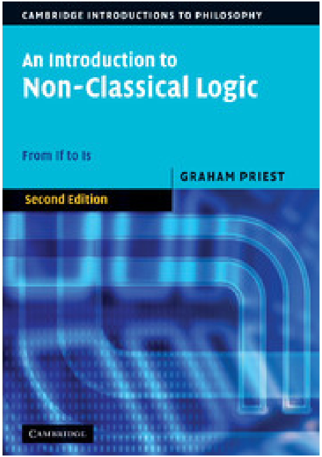


Non-Classical Logics

Some sources for non-classical logics

- Reject of the classic logic principles.
- Reduction of the classic logical constants.
- Expansion of the classical logical constants.
- Reject of the classical properties of the consequence relation.
- Modifications to the mathematical structure of the classical consequence relation.

Non-Classical Logics



Graham Priest (1948 -)

Non-Classical Logics

Notation

Set of well-formed formulae	$\mathcal{F}$
Formulae	$\alpha, \beta, \delta, \ldots$
Theories	$\Delta, \Gamma, \ldots$
Logical constants	$\neg, \wedge, \vee, \rightarrow, \bot$
Consequence relations	$\vdash, \models, \Vdash$

Reject of the Principle of Bivalence

Principle of bivalence

Every proposition is either true or false.

Reject of the Principle of Bivalence

Many-valued logics

The number of truth values is not restricted to only two. See, e.g. (Rescher 1969; Peña 1993).

- Truth values (Peña 1993, pp. 33-35)
  - designed
  - anti-designed
  - designed and anti-designed
  - neither designed nor anti-designed
  - no designed
  - no anti-designed

Reject of the Principle of Bivalence

Many-valued logics (continuation)

- Semantical universe (Peña 1993, p. 21)
  (i) 0: Minimal element, anti-designed and no designed.
  (ii) 1: Maximal element, designed and no anti-designed.
  (iii) $\forall\alpha\,(0 \leq |\alpha| \leq 1)$, where $|\alpha|$ is the truth-value of α and $\leq$ is a partial or total order.

Reject of the Principle of Bivalence

Example (Kleene's K_3 logic)

- Semantical universe

1	true	designed
½	undefined	anti-designed
0	false	anti-designed

Reject of the Principle of Bivalence

Example (Kleene's K_3 logic)

- Semantical universe

1	true	designed
½	undefined	anti-designed
0	false	anti-designed

- Truth tables

	$\neg$
1	0
½	½
0	1

$\wedge$	**1**	**½**	**0**
1	1	½	0
½	½	½	0
0	0	0	0

$\vee$	**1**	**½**	**0**
1	1	1	1
½	1	½	½
0	1	½	0

$\rightarrow$	**1**	**½**	**0**
1	1	½	0
½	1	½	½
0	1	1	1

Reject of the Principle of Bivalence

Example (continuation)

- A feature
  There is not α such that $\vDash_{\mathrm{K}_3} \alpha$.
- See, e.g. (Epstein 1990).

Reject of the Principle of Explosion

Principle of explosion (pseudo-Scotus, *ex contradictione sequitur quod libet)*

$$\forall \Gamma\ \forall \alpha\ \forall \beta\ (\Gamma, \alpha, \neg \alpha \vdash_{\mathrm{CL}} \beta).$$

Paraconsistent logics

$$\exists \Gamma\ \exists \alpha\ \exists \beta\ (\Gamma, \alpha, \neg \alpha \nVdash_{\mathrm{P}} \beta).$$

See, e.g. (Bobenrieth 1996) and (Carnielli and Marcos 2002).

Reject of the Principle of Explosion

Example (da Costa's C_1 logic)

- Bivalent semantic for C_1

  A valuation for C_1 is a function

$$v : \mathcal{F}(\mathrm{C}_1) \to \{0, 1\}$$

  such that:

  (i) $v(\alpha * \beta)$ has classical behavior $(* \in \{\wedge, \vee, \to\})$

  (ii) for negation

$$v(\alpha) = 0 \Rightarrow v(\neg\alpha) = 1,$$
$$v(\neg\neg\alpha) = 1 \Rightarrow v(\alpha) = 1.$$

Reject of the Principle of Explosion

Example (continuation)

- A consequence
  The semantic for C_1 is not truth-functionality:

$$v(\alpha) = 1 \nRightarrow v(\neg\alpha) = 1,$$
$$v(\alpha) = 1 \nRightarrow v(\neg\alpha) = 0.$$

- A feature
  The logic C_1 admits a strong negation

$$\sim \alpha \stackrel{\text{def}}{=} \neg\alpha \wedge \alpha^{\circ},$$

  where $^{\circ}$ is the well-behavior operator. The negation $\sim$ is a classical negation.
- See, e.g. (Marcos 1999, p. 47).

Reject of the Principle of the Excluded Third

Principle of the excluded third

$$\vdash_{\mathrm{CL}} \alpha \vee \neg\alpha, \text{ for all formula } \alpha.$$

Intuitionistic logics

- Computational meaning of the logical constants
- Propositions-as-types correspondence

See, e.g. (van Dalen 2013) and (Sørensen and Urzyczyn 2006).

Reject of the Principle of the Excluded Third

The Brouwer-Heyting-Kolmogorov (BHK) interpretation

A construction of	Consists of
$\alpha_1 \wedge \alpha_2$	A construction of α_1 and a construction of α_2.
$\alpha_1 \vee \alpha_2$	An indicator $i \in \{1, 2\}$ and a construction of α_i.
$\alpha_1 \to \alpha_2$	A method (function) which takes any construction of α_1 to a construction of α_2.
$\bot$	There is not construction.
$\neg\alpha \stackrel{\mathrm{def}}{=} \alpha \to \bot$	A method (function) which takes any construction of α into a nonexistent object.

Reject of the Principle of the Excluded Third

The Brouwer-Heyting-Kolmogorov (BHK) interpretation (continuation)

A construction of	Consists of
$\exists x \in U.\varphi(x)$	An element $a \in U$ and a construction of $\varphi(a)$.
$\forall x \in U.\varphi(x)$	A method (function) which takes any element $x \in U$ to a construction of $\varphi(x)$.

Reject of the Principle of the Excluded Third

Proofs by contradiction (or *reductio ad absurdum*) and proofs of negations

Proof by contradiction

$$\dfrac{\begin{array}{c}[\neg\beta]\\ \vdots \\ \bot\end{array}}{\beta}$$

Proof of negation (Bauer 2017)

$$\dfrac{\begin{array}{c}[\beta]\\ \vdots \\ \bot\end{array}}{\neg\beta}$$

Reject of the Principle of the Excluded Third

Justifications

Proof by contradiction

$$\dfrac{\dfrac{\dfrac{\begin{array}{c}[\neg\beta]\\ \vdots \\ \bot\end{array}}{\neg\beta \to \bot}\text{(conditional proof)}}{\neg\neg\beta}(\neg\alpha \stackrel{\text{def}}{=} \alpha \to \bot)}{\beta}(\vdash \neg\neg\alpha \to \alpha)$$

Proof of negation

$$\dfrac{\dfrac{\begin{array}{c}[\beta]\\ \vdots \\ \bot\end{array}}{\beta \to \bot}\text{(conditional proof)}}{\neg\beta}(\neg\alpha \stackrel{\text{def}}{=} \alpha \to \bot)$$

Reject of the Principle of the Excluded Third

Some features

- Since $\alpha \vee \neg\alpha$ and $\neg\neg\alpha \to \alpha$ are equivalents the proofs by contradiction are not accepted in intuitionistic logics.

Reject of the Principle of the Excluded Third

Some features

- Since $\alpha \vee \neg\alpha$ and $\neg\neg\alpha \to \alpha$ are equivalents the proofs by contradiction are not accepted in intuitionistic logics.
- The proofs of negations are intuitionistically valid.

Expansion of the Logical Constants

Example

Modal logics (Hughes and Cressivell 1998)

($\Box$: necessity, $\Diamond$: possibility)

- Temporal logics
- Epistemic logics
- Deontic logics

Reduction of the Logical Constants

Possible reductions

- Positive logics
- Implicative logics
- . . .

See, e.g. (Rasiowa 1974).

General question

What is a logical constant? (for example $\{\neg, \wedge, \vee, \rightarrow\}$)

A General Definition of Logic?

Definition

A logic $\mathfrak{L}$ is a structure $\mathfrak{L} = \langle \mathcal{F}, \Vdash \rangle$ where the consequence relation $\Vdash$ defined on $P(\mathcal{F}) \times \mathcal{F}$ satisfies (Carnielli and Marcos 2002; Gabbay 1994; Béziau 2000; Deakin and Shillito 2025):

$$\text{If } \alpha \in \Gamma, \text{ then } \Gamma \Vdash \alpha \qquad \text{(reflexivity)}$$

$$\text{If } \Gamma \Vdash \alpha \text{ and } \Gamma \subseteq \Delta, \text{ then } \Delta \Vdash \alpha \qquad \text{(monotony)}$$

$$\text{If } \Gamma \Vdash \alpha \text{ and } \Delta, \alpha \Vdash \beta, \text{ then } \Gamma, \Delta \Vdash \beta \qquad \text{(transitivity)}$$

$$\text{If } \Gamma \Vdash \alpha \text{ then } \sigma\Gamma \Vdash \sigma\alpha, \text{ for every substitution } \sigma \qquad \text{(structurality)}$$

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If $\alpha \in \Gamma,$ then $\Gamma \Vdash \alpha$	(reflexivity)
If $\Gamma \Vdash \alpha$ and $\Gamma \subseteq \Delta,$ then $\Delta \Vdash \alpha$	(monotony)
If $\Gamma \Vdash \alpha$ and $\Delta, \alpha \Vdash \beta,$ then $\Gamma, \Delta \Vdash \beta$	(transitivity)
If $\Gamma \Vdash \alpha$ then $\sigma\Gamma \Vdash \sigma\alpha,$ for every substitution σ	(structurality)

Remark

A 'Tarksian logic' is a logic whose consequence relation satisfies the above first three properties (Carnielli and Matulovic 2015). See also (Béziau 2005).

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Remark

The above definition was proposed by Tarski in 1930 and extended with substitution invariance in (Łos and Suszko 1958).

Reject of Properties of the Consequence Relation

Non-reflexivity logics

Alfabar logics (Krause and Béziau 1997). Schrödinger Logics (da Costa and Krause 1994). Weber (2017) says that (Strawson 1964) questions reflexivity.

Example

Let $\mathfrak{L} = \langle \mathcal{F}, \Vdash \rangle$ be a logic such that $\Gamma \Vdash \alpha$ iff exists Γ' such that

(i) $\Gamma' \subseteq \Gamma$,

(ii) Γ' is consistent and

(iii) $\Gamma' \vdash_{\mathrm{CL}} \alpha$.

Therefore, $p \wedge \neg p \nVdash p \wedge \neg p$ (Krause and Béziau 1997).

Reject of Properties of the Consequence Relation

Non-monotonic logics

‘*family of formal frameworks...in which reasoners draw conclusions tentatively, reserving the right to retract them in the light of further information.*’ (Strasser and Antonelli 2014)

Reject of Properties of the Consequence Relation

Non-transitive logics

Weber (2017) mentions some non-transitive logics by Smiley (1959) and Ripley. See also (Weir 2015) and (Ripley 2018).

Modifications to the Mathematical Structure of the Consequence Relation

Multiple consequence

$\Vdash \subseteq P(\mathcal{F}) \times P(\mathcal{F})$

Sub-structural logics

- Multi-set $\neq$ set: $(\{A, A, B\} \neq \{A, B\})$, therefore $\alpha, \alpha, \beta \Vdash \gamma$ does not imply $\alpha, \beta \Vdash \gamma$.
- $\alpha, \beta \Vdash \gamma$ does not imply $\beta, \alpha \Vdash \gamma$.
- In general, a theory Γ has not to be a set.

See, e.g. (Restall 2004).

Béziau's 'approach'

(i) Béziau (2000). 'What is Paraconsistent Logic?'

(ii) Béziau, de Freitas and Viana (2001). 'What is Classical Propositional Logic? (A Study in Universal Logic)'.

(iii) Béziau (2002). 'Are Paraconsistents Negations Negations?'

(iv) Béziau (2004). 'What Is The Principle of identity'.

(v) Béziau (2010). 'What Is a Logic. Towards Axiomatic Emptiness'.

(vi) Béziau (2022). 'Is Logic Exceptional'.

(vii) Béziau (2023). 'Why Logics'.

Universal Logic

Some questions

(i) Other approaches to the consequence relations (e.g. visual inference).
(ii) Equivalence criteria between semantics, syntax and algebra for a logic.
(iii) Equivalence criteria between logics (e.g. possible-translation semantics).
(iv) Minimal properties of the logical connectives (e.g. what is a negation?).
(v) Compatibility between the logical connectives.
(vi) High-order logic extensions.

Universal Logic

Universal logic is not itself a system of logic; it is a general study of the various systems of logic, considered as logical structures, in the same way that universal algebra is a general study of algebras considered as algebraic structures. Universal logic promotes unity in diversity not by reducing everything to one system but by developing concepts in a general framework to have a better understanding of the universe of logic systems. (Béziau 2023, p. 150)

Possible Applications

- Mathematical theories construction (Mortensen 1995).
- Hypercomputation (Sylvan and Copeland 1998; Agudelo and Sicard 2004)
- 'Or maybe paraconsistent logic will save us from the tricephalous CGC-monster (CGC for Cantor-Gödel-Church) by providing foundations for finite decidable complete mathematics.' (Béziau 1999, p. 16)

Conclusions

- Tolerance principle in Mathematics (Newton da Costa, 1958):

  '*Desde el punto de vista sintáctico-semántico, toda teoría es admisible, desde que no sea trivial. En sentido amplio, existe, en matemática, lo que no sea trivial.*' (Bobenrieth 1996, p. 180)

- Logical pluralism. See, e.g. (Bueno 2002).
- A new crisis? New opportunities?

A Category of Logics (Bonus Slides)

Definition

A category C is given by the following data:

- A class of objects $\mathrm{Obj}(C)$.
- A class of arrows or morphisms $\mathrm{Mor}(C)$.
- The functions $\mathrm{dom}, \mathrm{cod} : \mathrm{Mor}(C) \to \mathrm{Obj}(C)$.

  Notation:

  $$f : A \to B \equiv f \in \mathrm{Mor}(C), \quad \mathrm{dom}\, f = A, \quad \mathrm{cod}\, f = B.$$

- For $A \in \mathrm{Obj}(C)$, the identity arrow $\mathrm{id}_A : A \to A$.
- A composition operator $\circ : \mathrm{Mor}(C) \times \mathrm{Mor}(C) \to \mathrm{Mor}(C)$.

A Category of Logics

These data are subject to the follow conditions:

- $g \circ f$ is defined iff $\mathrm{cod}\ g = \mathrm{dom}\ f$.
- If $g \circ f$ is defined, then

$$\mathrm{dom}(g \circ f) = \mathrm{dom}\ f \quad \text{and} \quad \mathrm{cod}(g \circ f) = \mathrm{cod}\ g.$$

- For any $f : A \to B$,

$$\mathrm{id}_B \circ f = f \quad \text{and} \quad f \circ \mathrm{id}_A = f.$$

- For any $f : A \to B, g : B \to C, h : C \to D$,

$$h \circ (g \circ f) = (h \circ g) \circ f.$$

A Category of Logics

Example (The category **Set**)

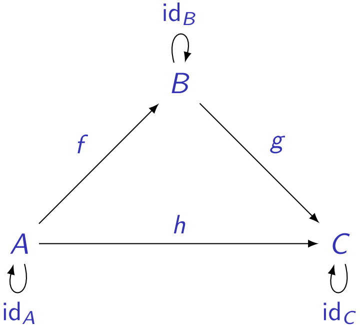
- $\mathrm{Obj}(\mathbf{Set})$: Sets
- $\mathrm{Mor}(\mathbf{Set})$: functions
- The identity arrow id_A: The identity function
- The composition operator $\circ$: The composition of functions

Technical remark

The usual definition of a function $f : A \to B$ as a set $f \subseteq A \times B$ which is single-valued and totally defined is not sufficient to uniquely determine $\mathrm{cod}\, f$. Therefore it is necessary to define f as a triple $(A, \mathrm{graph}(f), B)$.

A Category of Logics

Example (The category **3**)



A Category of Logics

Example

Almost every known example of a mathematical structure with the appropriate structure-preserving map yields a category.

Category	Objects	Morphisms
Set	Sets	Functions
Pfn	Sets	Partial functions
Vect	Vector spaces	Linear transforms
Top	Topological spaces	Continuous functions
Poset	Posets	Monotone functions
CPO	Complete posets	Continuous functions
Lat	Lattices	Structure preserving homomorphisms

A Category of Logics

Example

A deductive system $\Vdash_D$ can be turned on a category $\mathbf{D}$

- $\mathrm{Obj}(\mathbf{D})$: Formulae
- $\mathrm{Mor}(\mathbf{D})$: Proofs
- The identity arrow $\mathrm{id}_A : A \to A$: A proof of $A \Vdash_D A$
- The composition operator $\circ$: Transitivity of the $\Vdash_D$

$$\frac{f : A \to B \qquad g : B \to C}{g \circ f : A \to C}$$

References

Agudelo, Juan C. and Sicard, Andrés (2004). Máquinas de Turing Paraconsistentes: Una Posible Definición. Matemáticas: Enseñanza Universitaria XII.2, pp. 37–51 (cit. on p. 34).

Bauer, Andrej (2017). Five States of Accepting Constructive Mathematics. Bulletin of the American Mathematical Society 54.3, pp. 481–498. DOI: 10.1090/bull/1556 (cit. on p. 18).

Béziau, Jean-Yves (1999). The Future of Paraconsistent Logic. Logical Studies 2 (cit. on p. 34).

— (2000). What is Paraconsistent Logic? In: Frontiers of Paraconsistent Logic. Ed. by Batens, Diderik et al. Hertfordshire, England: Research Studies Press LTD., pp. 95–111 (cit. on pp. 24–26, 31).

— (2002). Are Paraconsistents Negations Negations? In: Paraconsistency. The Logical Way to the Inconsistent. Ed. by Carnielli, Walter A., Coniglio, Marcello E. and D'Ottaviano, Ítala M. L. Marcel Dekker, pp. 465–486 (cit. on p. 31).

— (2005). From Consequence Operator to Universal Logic: A Survey of General Abstract Logic. In: Logica Universalis. Ed. by Béziau, Jean-Yves. Birkhäuser Basel, pp. 3–17. DOI: 10.1007/978-3-7643-8354-1_1 (cit. on pp. 24–26).

References

Béziau, Jean-Yves (2023). Why Logics. Logics 1, pp. 148–156. DOI: 10.3390/logics1030007 (cit. on pp. 31, 33).

Béziau, Jean-Yves, de Freitas, Renata Pereira and Viana, Jorge Petrucio (2001). What is Classical Propositional Logic? (A Study in Universal Logic). Logical Studies 7 (cit. on p. 31).

Bobenrieth, M. Andrés (1996). ¿Inconsistencias, Por Qué No? Santafé de Bogotá: Tercer Mundo Editores, División Gráfica (cit. on pp. 12, 35).

Bueno, Otávio (2002). Can a Paraconsistent Theorist be a Logical Monist? In: Paraconsistency. The Logical Way to the Inconsistent. Ed. by Carnielli, Walter A., Coniglio, Marcello E. and D'Ottaviano, Ítala M. L. Marcel Dekker, pp. 535–552 (cit. on p. 35).

Carnielli, Walter and Matulovic, Mariana (2015). The Method of Polynomial Ring Calculus and its Potentialities. Theoretical Computer Science 606.42–56. DOI: 10.1016/j.tcs.2015.05.015 (cit. on pp. 24–26).

Carnielli, Walter A., Coniglio, Marcello E. and D'Ottaviano, Ítala M. L., eds. (2002). Paraconsistency. The Logical Way to the Inconsistent. Marcel Dekker.

References

Carnielli, Walter A. and Marcos, João (2002). A Taxonomy of C-systems. In: Paraconsistency. The Logical Way to the Inconsistent. Ed. by Carnielli, Walter A., Coniglio, Marcello E. and D'Ottaviano, Ítala M. L. Marcel Dekker, pp. 1–94 (cit. on pp. 12, 24–26).

da Costa, Newton C. A. and Krause, Décio (1994). Schrödinger Logics. Studia Logica 53.4, pp. 533–550. DOI: 10.1007/BF01057649 (cit. on p. 27).

Deakin, Jonte and Shillito, Ian (2025). Bi-Intuitionistic Logics through the Abstract Algebraic Logic Lens. URL: https://arxiv.org/abs/2503.17159 (cit. on pp. 24–26).

Epstein, Richard L. (1990). The Semantic Foundations of Logic. Vol. 1. Propositional Logics. Kluwer Academic Publishers (cit. on p. 11).

Gabbay, D. M. (1994). What is a Logic System? In: What is a Logical System? Ed. by Gabbay, D. M. Clarendon Press. Chap. 7 (cit. on pp. 24–26).

Hughes, G. E. and Cressivell, M. J. (1998). A New Introduction to Modal Logic. Routledge (cit. on p. 22).

Krause, Décio and Béziau, Jean-Yves (1997). Relativizations of the Principle of Identity. Logic Journal of the IGPL 5.3, pp. 327–338. DOI: 10.1093/jigpal/5.3.1-b (cit. on p. 27).

References

Łos, J. and Suszko, R. (1958). Remarks on Sentential Logics. Indagationes Mathematicae (Proceedings) 61, pp. 177–183. DOI: 10.1016/S1385-7258(58)50024-9 (cit. on pp. 24–26).

Marcos, João (1999). Semânticas de Traduções Possíveis. MA thesis. U. Estadual de Campinas (cit. on p. 14).

Mortensen, Chris (1995). Inconsistent Mathematics. Kluwer Academic Publishers (cit. on p. 34).

Peña, Lorenzo (1993). Introducción a las Lógicas No Clásicas. Instituto de Investigaciones Filosóficas. Colección: Cuadernos. Universidad Autónoma de México (cit. on pp. 7, 8).

Rasiowa, Helena (1974). An Algebraic Approach to Non-Classical Logics. Vol. 78. Studies in Logic and the Foundations of Mathematics. North-Holland Publishing Company (cit. on p. 23).

Rescher, Nicholas (1969). Many-Valued Logic. McGraw-Hill (cit. on p. 7).

Restall, Greg (2004). An Introduction to Substructural Logics. Taylor & Francis (cit. on p. 30).

Ripley, David (2018). On the 'Transitivity' of Consequence Relations. Journal of Logic and Computation 28.2, pp. 433–450. DOI: 10.1093/logcom/exx039 (cit. on p. 29).

Smiley, T. J. (1959). Entailment and Deducibility. Proceedings of the Aristotelian Society 59.1, pp. 233–254. DOI: 10.1093/aristotelian/59.1.233 (cit. on p. 29).

References

- Sørensen, Morten-Heine and Urzyczyn, Paul (2006). Lectures on the Curry-Howard Isomorphism. Vol. 149. Studies in Logic and the Foundations of Mathematics. Elsevier (cit. on p. 15).
- Strasser, Christian and Antonelli, G. Aldo (2014). Non-Monotonic Logic. Ed. by Zalta, Edward N. The Stanford Encyclopedia of Philosophy (cit. on p. 28).
- Strawson, P. F. [1952] (1964). Introduction to Logical Theory. Reprinted. Metheun & Co LTD (cit. on p. 27).
- Sylvan, Richard and Copeland, Jack (1998). Computability is Logic-Relative. Philosophy Research Papers, University of Canterbury 5, pp. 1–16 (cit. on p. 34).
- van Dalen, Dirk [1980] (2013). Logic and Structure. 5th ed. Springer (cit. on p. 15).
- Weber, Zach (2017). A Guide to Logical Pluralism for Non-Logicians. Think 16.47, pp. 93–114. DOI: 10.1017/S1477175617000239 (cit. on pp. 27, 29).
- Weir, Alan (2015). A Robust Non-Transitive Logic. Topoi 34.1, pp. 99–107. DOI: 10.1007/s11245-013-9176-9 (cit. on p. 29).