A process calculus for spatially-explicit ecological models

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Background

- One trend of **theoretical ecology**: Individual-based modeling of ecosystems.
- **Individual-based modeling** is the opposite to population-based modeling
- Application area: **Metapopulations**
  - Local populations in spatially-separated habitat patches
  - Populations interact locally inside a patch
  - Individuals can disperse among patches
- Conservation ecology, species reintroduction

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  - Differential equations
  - Recurrence equations
- **Formal methods** individual-based modeling of ecological systems
  - Process calculi, P-systems, cellular automata

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- Simulations carried out by ecologists often impose an order on the events that may take place within a model
- Ordering can have implications on the simulation
- Examples of temporal **process ordering** in ecological systems
    - Concurrent ordering
    - Reproduction before mortality
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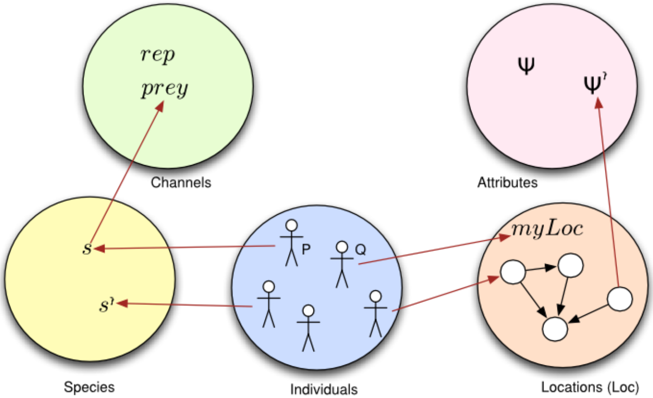
Our contributions

- Process Algebra with Locations for Population Systems (PALPS)
  - spatial calculus, locations, location attributes
  - location dependent behavior of individuals
  - **Process ordering** as a **policy**
  - semantics for a **policy** for actions
  - formal translation to model checker PRISM
  - simulation results

PALPS

- Basic entities
  - Individuals, Species, Locations, Channels and Attributes



PALPS

- Examples of expressions
  - There is only one individual of species **s** in myloc:
    $\mathbf{s}@\text{myloc} = 1$
  - Temperature is less than 40 or Humidity is higher that 90 at location ℓ:
    $T@\ell > 40 \vee H@\ell > 90$
  - Total number of individuals at location ℓ:
    $\mathbf{s}@\text{myloc} + \mathbf{s}'@\text{myloc} < 10$

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PALPS syntax (1)

- The individual level

$$
\begin{array}{rcll}
P & ::= & \mathbf{0} & \text{inactive individual} \\
 & | & \sum_{i\in I} \eta_i.P_i & \text{non-deterministic choice} \\
 & | & \bullet\!\!\sum_{i\in I} p_i{:}P_i & \text{probabilistic choice} \\
 & | & \text{cond}\,(e_1 \rhd P_1,\ldots,e_n \rhd P_n) & \text{conditional} \\
 & | & C & \text{constant}
\end{array}
$$

- Actions

$$\eta \quad ::= \quad a \mid \bar{a} \mid go\ \ell \mid \surd \qquad \text{input,output,move,time}$$

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$$R \;::=\; !rep.P$$

- The system level

S	$::=$	$\mathbf{0}$	inactive system
	$\mid$	$P{:}\langle \mathbf{s}, \ell \rangle$	located individuals
	$\mid$	$R{:}\langle \mathbf{s} \rangle$	named species
	$\mid$	$S_1 \mid S_2$	parallel composition
	$\mid$	$S \backslash L$	restriction

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PALPS semantics (1)

- Operational semantics defined at the level of configurations (E, S)
  - E: an environment
  - S: a population system
- The environment in needed to evaluate the expressions
- As an example, the initial environment for

$$S \stackrel{\text{def}}{=} (P_0{:}\langle \ell, \mathbf{s}, 2\rangle | P_0{:}\langle \ell', \mathbf{s}\rangle | (!rep.P_0){:}\langle \mathbf{s}\rangle)\backslash\{rep\}.$$

is

$$E \stackrel{\text{def}}{=} \{(\ell, \mathbf{s}, 2), (\ell', \mathbf{s}, 1)\}$$

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PALPS semantics (2)

- Two transition relations
  - Probabilistic transition relation

$$(E, S) \xrightarrow{w}_p (E', S')$$

  - Non-deterministic transition relation

$$(E, S) \xrightarrow{\alpha}_n (E', S')$$

PALPS semantics (3)

- The semantics is given at two levels
  - Individual level
  - System level
- Asynchronous communication
- All processes synchronize on the time passing actions

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Process ordering in PALPS (1)

- A **policy** σ is a partial order on the set of PALPS non-probabilistic actions.
- A policy is set of tuples (α, β), where α, β are actions
- A policy models **process ordering** in ecological systems

Process ordering in PALPS (2)

- A prioritized transition relation

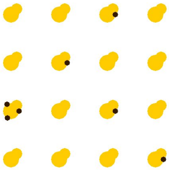
$$\frac{(E,S) \xrightarrow{\alpha}_n (E',S') \text{ and } (E,S) \not\xrightarrow{\beta}_n, (\alpha,\beta) \in \sigma}{(E,S) \xrightarrow{\alpha}_\sigma (E',S')}$$

Process ordering in PALPS (3)

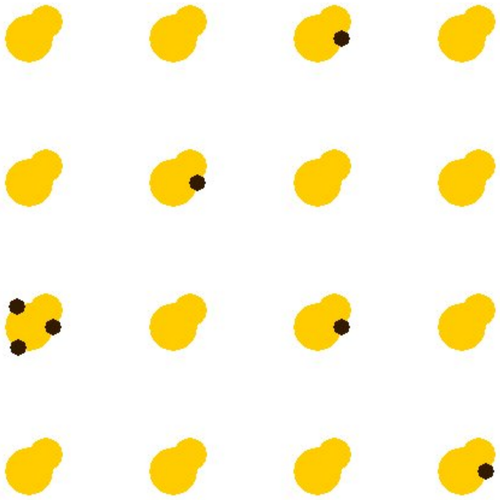
- Examples of policies in PALPS. Let $\ell, \ell' \in \mathbf{Loc}$,
  - Concurrent ordering $\sigma = \{\}$
  - Reproduction before dispersal $\sigma = \{(\tau_{rep,\ell,\mathbf{s}},\ \tau_{go,\ell',\mathbf{s}})\}$
  - Dispersal before reproduction $\sigma = \{(\tau_{go,\ell',\mathbf{s}},\ \tau_{rep,\ell,\mathbf{s}})\}$

PALPS example (1)

- Varroa-mite parasites live on an $n \times n$ lattice of honey-bee cells and cycle through the following.
  - **Death**: with probability p
  - **Dispersal**: randomly
  - **Reproduction**: produces an offspring of size b



PALPS example (2)



PALPS example (3)

- The individual level

$$P_0 \stackrel{\text{def}}{=} p{:}P_1 + (1-p){:}\surd.0$$

$$P_1 \stackrel{\text{def}}{=} \sum_{\ell \in \mathbf{Nb}(\text{myloc})} \frac{1}{4} : go\,\ell.$$

$$\text{cond}\,(\mathbf{s}@\text{myloc} = 1 \triangleright P_2;\ \text{true} \triangleright \surd.0)$$

$$P_2 \stackrel{\text{def}}{=} \overline{rep}^b.\surd.0$$

$$\text{where } \overline{rep}^b \stackrel{\text{def}}{=} \underbrace{\overline{rep}\ldots\overline{rep}}_{b \text{ times}}$$

PALPS example (4)

- The species level

$$R \stackrel{\text{def}}{=} !rep.P_0$$

- The system level

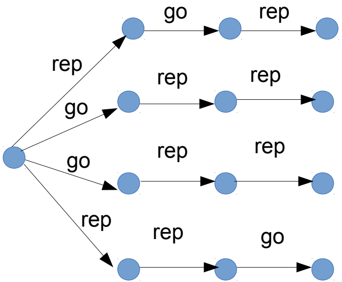
$$System \stackrel{\text{def}}{=} (P_0{:}\langle \ell, \mathbf{s}, 2\rangle | P_0{:}\langle \ell', \mathbf{s}\rangle | (!rep.P_0){:}\langle \mathbf{s}\rangle)\backslash\{rep\}.$$

PALPS example (5)

- We use the policy **dispersal before reproduction** $\{(\tau_{rep,\ell,\mathbf{s}}, \tau_{go,\ell',\mathbf{s}}) | \ell, \ell' \in \mathbf{Loc}\}$ for this example.

PALPS example (6)

- Semantics of the policy **dispersal before reproduction** $\{(\tau_{rep,\ell,\mathbf{s}}, \tau_{go,\ell',\mathbf{s}})\}$ for the example.



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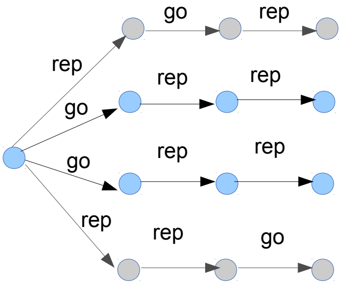

Encoding of PALPS into PRISM (1)

- PRISM is a probabilistic model checker[1]
- To translate PALPS into the PRISM language
  - each process is a module
  - the execution flow is captured by a local variable
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[1] www.prismmodelchecker.org/

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Encoding of PALPS into PRISM (2)

- To translate PALPS into the PRISM language
  - we map binary communication into multi-way communication
  - replication is bounded
  - we define a global variable for each action to ensure the semantics of the policy

Encoding of PALPS into PRISM (3)

Correctness

For any configuration (E, Sys) and policy σ, where E is compatible with Sys, whenever $(E, Sys) \stackrel{\alpha}{\longrightarrow} (E', Sys')$ then $[\![(E, Sys)]\!] \longrightarrow^m [\![(E', Sys')]\!]$ where $1 \leq m \leq 3$.

- A similar result holds in the opposite direction.

Model checking of PALPS using PRISM (1)

- Verification of probabilistic temporal PCTL properties
  - Probability of extinction of the population in the next 10 years is less than a certain threshold p_e
  - Within the next 20 years with some high probability, members of the population **s** will outnumber the members of population **s**′
  - Compare the average number of individuals of species s at time unit t to a constant

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- Semantics of model checking
  - **Defined over Markov Decision Processes:** Computes minimum and maximum probabilities
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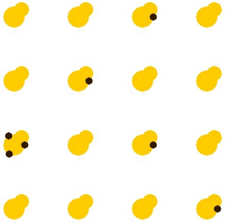
Simulation of PALPS using PRISM

- Explore random paths of execution
- Search for deadlocks using PRISM simulation
- Perform **model-checking by simulation**



Results for the example (1)

Case study size	Number of States	Construction time (sec.)	RAM (GB)
No policy			
3 PALPS individuals 4 PALPS individuals	130397 1830736	8 101	0.5 1.9
Policy σ			
3 PALPS individuals 4 PALPS individuals	27977 148397	3 10	0.3 0.7
Extended policy			
3 PALPS individuals 4 PALPS individuals	20201 128938	3 9	0.3 0.6

Table : Performance of building probabilistic models in PRISM with and without policies.

Results for the example (2)

- Applying a policy $\sigma = \{(\tau_{rep,\ell,\mathbf{s}}, \tau_{go,\ell',\mathbf{s}}) | \ell, \ell' \in \mathbf{Loc}\}$ reduced the size of the state space by a factor of 10
- Applying a policy for the execution of actions among individuals reduced the state space by about 20% more

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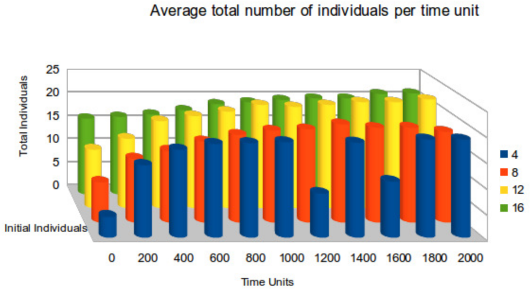
Results for the example (3)

- Results obtained using statistical model checking
  - Using simulation to verify a PCTL property

Results for the example (4)

- Expected population size vs simulation time for different initial sizes of the population, with offspring size $b = 2$.



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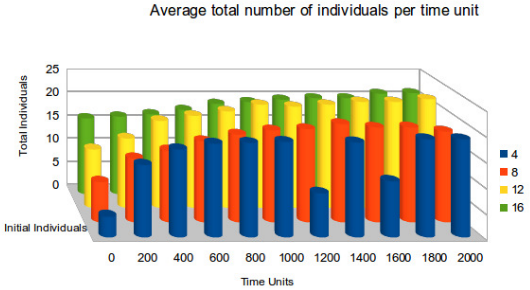

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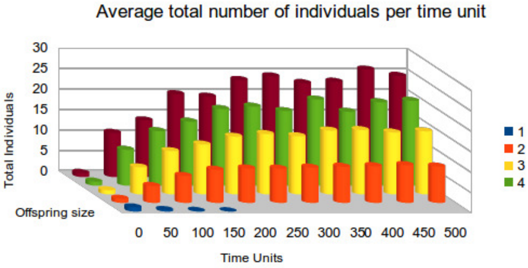

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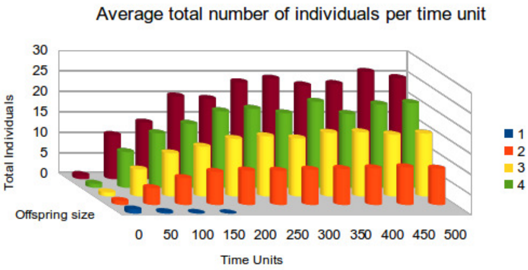

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Reducing the state space (1)

- We reduced the state space of PALPS models with policies, but
  - for some applications, it is still too big
- Proposed solution
  - Synchronous communication [3]
  - Mean-field semantics

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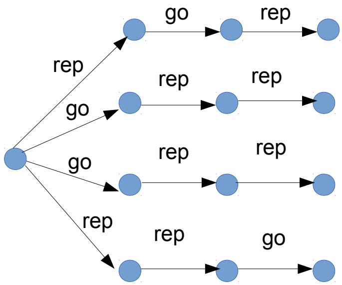
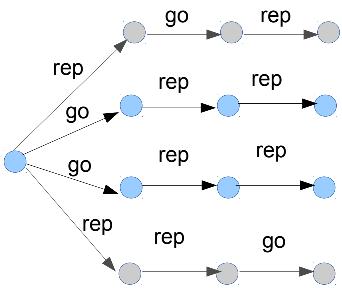
Reducing the state space (2)

- Complete state space



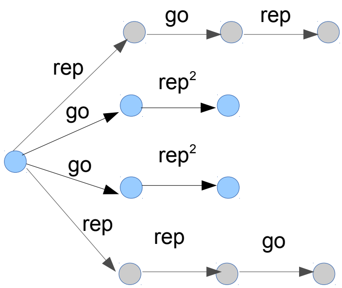
Reducing the state space (3)

- State-space reduced with a policy



Reducing the state space (4)

- State-space reduced with synchronous communication

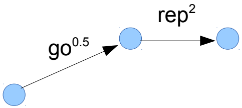


Conclusions

- PALPS
  - Discrete space, discrete time, probabilistic behavior
  - Location attributes and location-dependent behavior
  - **Policies** that
    - **Reduce the state space**
    - **Allow to model different process orderings**
  - Semantics for PALPS with synchronous communication
  - Support for simulation and analysis of models through PRISM translation

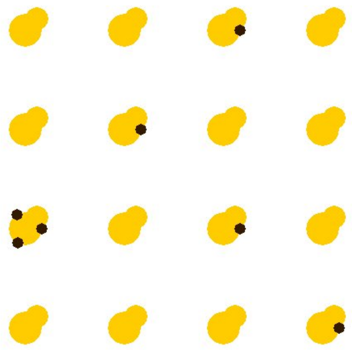
Future work

- Mean-field semantics à la WSCCS



- T H A N K Y O U

  Do you have any question?



References

1. A. Philippou, M. Toro-Bermdez and Margarita Antonaki. PALPS: A process calculus for spatially-explicit ecological models. Scientific Annals of Computer Science, 23(1):119–167, 2013.
2. A. Philippou and M. Toro-Bermdez. Process ordering in a process calculus for spatially-explicit ecological models. In Proc. of MokMasd '13, Madrid, Spain, September, 2013. Volume 8368 of Lecture Notes in Computer Science, pages 345–361, Springer July 2013.
3. M. Toro-Bermdez, A. Philippou, C. Kassara, S. Sfenthourakis. Synchronous Parallel Composition in a Process Calculus for Ecological Models. In Proc. of ICTAC 2014. Volume of Lecture Notes in Computer Science, 424–441, Springer October 2014.