Ordinals and Typed Lambda Calculus

Course Introduction

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Administrative Information

Course web page

`https://asr.github.io/courses/ordinals-and-typed-lambda-calculus/`

Lectures dates, source code, etc.

See course web page.

Informally Building Sets

Definition

A set is **pure** iff its members are also sets.

Notation

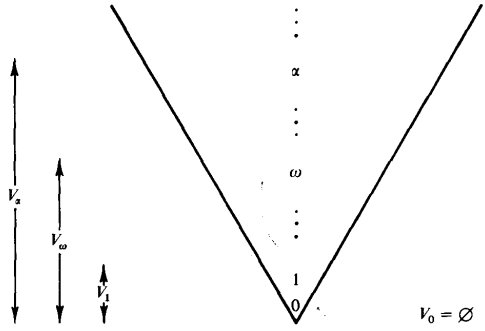
Let A be a set. The power set of A is denoted by $\mathrm{P}A$.

Convention

We shall use the terms ‘ordinal number’ and ‘ordinal’ interchangeably.

Informally Building Sets

The ordinal numbers are the backbone of the universe of (pure) sets*



$$V_0 \coloneqq \emptyset \qquad V_\omega \coloneqq V_0 \cup V_1 \cup \cdots \qquad V_{\alpha+1} \coloneqq \mathrm{P}V_\alpha$$
$$V_{n+1} \coloneqq \mathrm{P}V_n \qquad V_{\omega+1} \coloneqq \mathrm{P}V_\omega$$

*Figure source: [End1977, Fig. 3].

Classifying the Ordinals

- ordinals
  - zero ordinal
  - successor ordinal
  - limit ordinal

- ordinals
  - countable
    - computable (λ-definable or constructive)
    - incomputable
  - uncountable

References

[End1977] Herbert B. Enderton. Elements of Set Theory. Academic Press, 1977 (cit. on p. 4).