MT8002 Elements of Set Theory
List of Axioms

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Preliminaries

Textbook

Derek Goldrei (1996). Classic Set Theory. A Guided Independent Study.

Conventions

The numbers and page numbers assigned to chapters, examples, exercises, figures, quotes, sections and theorems on these slides correspond to the numbers assigned in the textbook.

See the appendix in the course homepage for the logical symbols, mathematical symbols and acronyms not defined in these slides.

Outline

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Classification

- Axioms stating the existence of sets
- Axioms determining properties of sets
- Axioms for building sets from other sets

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Axioms Stating the Existence of Sets

Axiom of Empty Set

$\exists x \forall y \; y \notin x$.

"*There is a set with no elements.*" *(p. 76)*

Axiom of Infinite

$\exists x \, (\emptyset \in x \wedge \forall y \, (y \in x \rightarrow y \cup \{ y \} \in x))$.

"*There is an inductive set.*" *(p. 92)*

Outline

Axioms Determining Properties of Sets

Axiom of Extensionality

$\forall x \forall y \, (x \doteq y \leftrightarrow \forall z \, (z \in x \leftrightarrow z \in y))$.

"*Two sets are equal if and only if they contain the same elements.*" *(p. 76)*

Axiom of Foundation

$\forall x \exists y \, (y \in x \wedge x \cap y \doteq \emptyset)$.

"*Every set is well-founded, i.e., contains an* $\in$*-minimal element.*" *(p. 92)*

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Axioms for Building Sets From Other Sets

Axiom of Separation

$\forall x \exists y \forall z (z \in y \leftrightarrow (z \in x \wedge \phi(z)))$, where $\phi(z)$ is a formula (possibly referring to named sets (possibly with parameters)) with free variable z.

"*For any set x there is a set consisting of all z in x for which $\phi(z)$ holds.*" *(p. 82)*

Axiom of Pairs

$\forall x \forall y \exists z \forall w (w \in z \leftrightarrow (w \doteq x \vee w \doteq y))$.

"*For any two sets, there is a set whose elements are precisely these sets.*" *(p. 76)*

Axiom of Union

$\forall x \exists y \forall z (z \in y \leftrightarrow \exists w (z \in w \wedge w \in x))$.

"*For any set x there is a set which is the union of all the elements of x.*" *(p. 82)*

Axioms for Building Sets From Other Sets

Axiom of Power Set

$\forall x \exists y \forall z (z \in y \leftrightarrow z \subseteq x)$.

"*For any set x there is a set consisting of all subsets of x.*" *(p. 82)*

Axiom of Replacement

$\forall x \exists y \forall y' (y' \in y \leftrightarrow \exists x' (x' \in x \wedge \phi(x', y')))$, where $\phi(s, t)$ is a formula (possibly referring to named sets (possibly with parameters)) such that $\forall s \exists t (\phi(s, t) \wedge \forall t' (\phi(s, t') \rightarrow t' \doteq t))$.

"*If $\phi(s, t)$ is a class function, then when its domain is restricted to x, the resulting images form a set y.*" *(p. 92)*

Outline

The Axiom of Choice

Axiom of Choice

"*Suppose that $\mathscr{F}$ is a family of non-empty sets. Then there is a function $h: \mathscr{F} \to \bigcup \mathscr{F}$ such that for each $A \in \mathscr{F}$, $h(A) \in A$. [The function] h is said to be a choice function for $\mathscr{F}$.*" *(p. 107)*

Outline

References

Derek Goldrei (1996). Classic Set Theory. A Guided Independent Study. Chapman & Hall (cit. on p. 2).