

CM0832 Elements of Set Theory

8. Ordinals and Order Types

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Preliminaries

Convention

The number assigned to chapters, examples, exercises, figures, sections, and theorems on these slides correspond to the numbers assigned in the textbook [Enderton 1977].

Ordinal Addition

Definition

Let α and β be ordinals. We define their **addition** by transfinite recursion on β :

$$\alpha + 0 = \alpha,$$

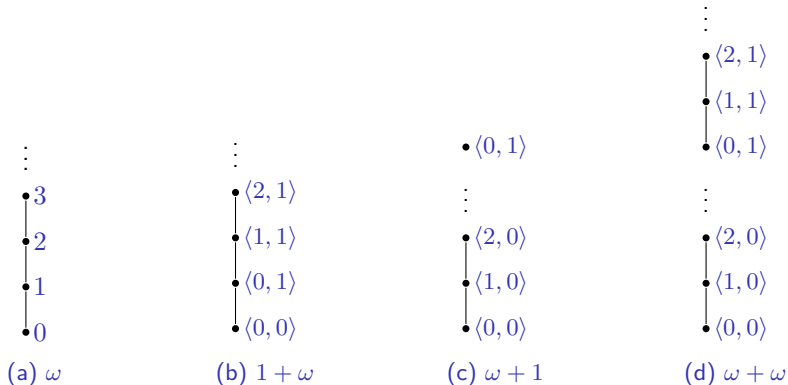
$$\alpha + \beta^+ = (\alpha + \beta)^+,$$

$$\begin{aligned}\alpha + \beta &= \sup \{ \alpha + \lambda \mid \lambda < \beta \} \\ &= \bigcup_{\lambda < \beta} (\alpha + \lambda), \text{ for all limit ordinal } \beta.\end{aligned}$$

Ordinal Addition

Example

$\omega = 1 + \omega$, $1 + \omega \neq \omega + 1$ and $\omega + 1 \neq \omega + \omega$.



References



Enderton, Herbert B. (1977). Elements of Set Theory. Academic Press (cit. on p. 2).