CM0832 Elements of Set Theory
List of Axioms

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Preliminaries

Convention

The number assigned to chapters, examples, exercises, figures, sections, and theorems on these slides correspond to the numbers assigned in the textbook [Enderton 1977].

List of Axioms

Axioms stating the existence of sets

- **Empty (existence) axiom**: There is a set having no members, that is,

$$\exists B \, \forall x \, (x \notin B).$$

- **Infinity axiom**: There exists an inductive set, that is,

$$\exists A \, [\, \emptyset \in A \wedge \forall a \, (a \in A \to a^{+} \in A) \,].$$

List of Axioms

Axioms determining properties of sets

- **Extensionality axiom**: If two sets have exactly the same members, then they are equal, that is,

$$\forall A\, \forall B\, [\, \forall x\, (x \in A \leftrightarrow x \in B) \rightarrow A = B\,].$$

- **Regularity (foundation) axiom**: All sets are well-founded, that is,

$$\forall A\, [\, A \neq \emptyset \rightarrow \exists m\, (m \in A \wedge m \cap A = \emptyset)\,].$$

List of Axioms

Axioms for building sets from other sets

- **Pairing axiom**: For any sets u and v, there is a set having as members just u and v, that is,

$$\forall a\,\forall b\,\exists C\,\forall x\,(x \in C \leftrightarrow x = a \vee x = b).$$

- **Union axiom** (first version): For any sets a and b, there is a set whose members are those sets belonging either to a or to b (or both), that is,

$$\forall a\,\forall b\,\exists B\,\forall x\,(x \in B \leftrightarrow x \in a \vee x \in b).$$

- **Union axiom** (final version): For any set A, there exists a set B whose elements are exactly the members of the members of A, that is,

$$\forall A\,\exists B\,\forall x\,[\,x \in B \leftrightarrow \exists b\,(x \in b \wedge b \in A)\,].$$

List of Axioms

Axioms for building sets from other sets (continuation)

- **Power set axiom**: For any set a, there is a set whose members are exactly the subsets of a, that is,
$$\forall a \, \exists B \, \forall x \, (x \in B \leftrightarrow x \subseteq a).$$
- **Subset axiom scheme** (axiom scheme of comprehension or separation): For any propositional function $\varphi(x, t_1, \ldots, t_k)$, not containing B, the following is an axiom:
$$\forall t_1 \cdots \forall t_k \, \forall c \, \exists B \, \forall x \, (x \in B \leftrightarrow x \in c \wedge \varphi(x, t_1, \ldots, t_k)).$$
- **Axiom of choice** (a version): For any relation R there is a function $F \subseteq R$ with $\operatorname{dom} F = \operatorname{dom} R$.

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List of Axioms

Axioms for building sets from other sets (continuation)

- **Replacement axiom scheme**: For any propositional function $\varphi(x, y, t_1, \ldots, t_k)$, not containing B, the following is an axiom:

$$\forall t_1 \cdots \forall t_k \, \forall A \, [\, \forall x \, (x \in A \to \exists! y \, \varphi(x, y, t_1, \ldots, t_k)) \to \\ \exists B \, \forall y \, (y \in B \leftrightarrow \exists x \, (x \in A \wedge \varphi(x, y, t_1, \ldots, t_k))) \,].$$

References

Enderton, Herbert B. (1977). Elements of Set Theory. Academic Press (cit. on p. 2).