CM0832 - MT5001 Elements of Set Theory
Introduction

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Universidad EAFIT

Semester 2026-2

Outline

Outline

Pacto Pedagógico

Como miembros de la Universidad EAFIT, nos comprometemos a actuar de manera íntegra siguiendo los más altos estándares éticos y morales.

- Respeto
- Tolerancia
- Honradez
- Compromiso

Pacto Pedagógico

Página web del curso

`https://asr.github.io/courses/cm0832-set-theory/2026-2/`

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Conducto regular, fechas y porcentajes de las evaluaciones

La información está en la página web del curso.

Pacto Pedagógico

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Conducto regular, fechas y porcentajes de las evaluaciones

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Responsabilidad compartida

- Profesor
- Estudiantes

Pacto Pedagógico

Asistencia a clase

Reglamento académico de los programas de posgrado, Título II, Capítulo VI, Artículo 62, Parágrafo 2.

> *"El estudiante de posgrado cuyas faltas de asistencia lleguen al treinta por ciento (30%) del total de las horas de clase programadas para el curso o para una parte de éste, cuando se desarrolle con más de un profesor, en secciones temáticas denominadas 'módulo', pierde con calificación de cero punto cero (0.0) del seminario o curso correspondiente y esta nota afecta el promedio crédito acumulado."*

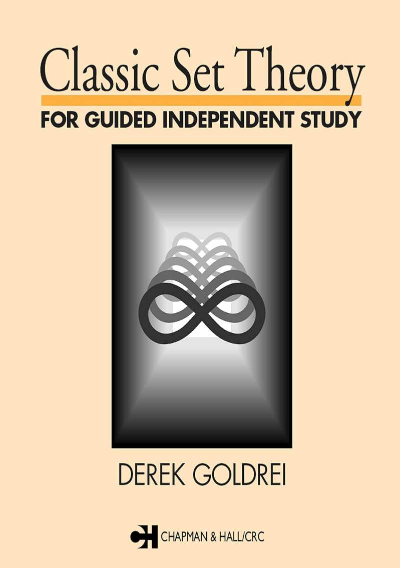
Pacto Pedagógico

Orientaciones para el curso

- Se recomienda cuatro horas de trabajo por semana (dos horas por cada hora de clase).
- Las clases son presenciales.
- La evaluación a la docencia es obligatoria.
- Se recomienda revisar periódicamente los canales de comunicación institucionales (EAFIT Interactiva y el correo institucional).
- El estudiante podrá entrar a clase a más tardar 20 minutos después de la hora asignada para su inicio.

Outline

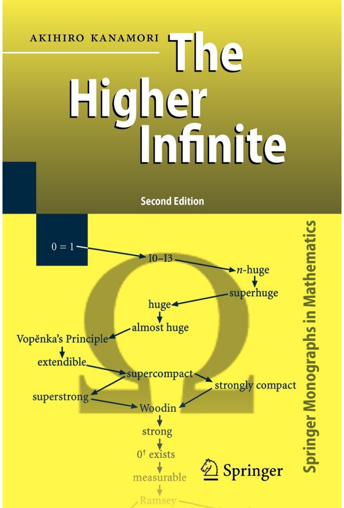
Textbook


Classic Set Theory
FOR GUIDED INDEPENDENT STUDY
DEREK GOLDREI
CHAPMAN & HALL/CRC

Course Outline

- Introduction. Formal languages and theories. Methods of proof.
- The Zermelo–Fraenkel axioms.
- The axiom of choice.
- Ordered sets.
- Ordinal numbers.

Birth



"Set theory was born on that December 1873 day when Cantor established that the reals are uncountable, i.e. there is no one-to-one correspondence between the reals and the natural numbers." (Kanamori [1994] 2009, p. XII)

Naive Set Theory

Remark

Cantor's set theory is also called naive set theory.

Cantor's set definition

*"Unter einer **Menge** verstehen wir jede Zusammenfassung M von bestimmten wohlunterschiedenen Objecten m unsrer Anschauung oder unseres Denkens (welche die **Elemente** von M genannt werden) zu einem Ganzen."*
(Cantor 1895, p. 481)

*"By an **aggregate** (Menge) we are to understand any collection into a whole M of definite and separate objects m of our intuition or our thought. These objects are called the **elements** of M." (Cantor [1895] 1915, p. 85)*

*"A **set** is a collection into a whole of definite, distinct objects of our intuition or our thought. The objects are called **elements (members)** of the set."*
(Hrbacek and Jech [1978] 1999, p. 1)

Naive Set Theory

Transfinite numbers

Cantor introduced two kinds of transfinite numbers, each possessing its own arithmetic system that builds upon and expands the conventional arithmetic of natural number: Ordinal and cardinal numbers.

The Crisis in the Foundations of Mathematics

$$\text{Paradoxes} \Rightarrow \text{Crisis} \Rightarrow \begin{cases} \text{Logicism} & \text{(Russell and Whitehead)} \\ \text{Formalism} & \text{(Hilbert)} \\ \text{Intuitionism} & \text{(Brouwer)} \end{cases}$$

The Crisis in the Foundations of Mathematics

Logicism (Russell and Whitehead)

> *"The logicistic thesis is that mathematics is a branch of logic. The mathematical notions are to be defined in terms of the logical notions. The theorems of mathematics are to be proved as theorems of logic." (Kleene [1952] 1974, p. 43)*

Formalism (Hilbert)

> *"Classical mathematics shall be formulated as a formal axiomatic theory, and this theory shall be proved to be consistent, i.e. free from contradiction."*
> *(Kleene [1952] 1974, p. 53)*

Intuitionism (Brouwer)

> *"Intuitionism is based on the idea that mathematics is a creation of the mind. The truth of a mathematical statement can only be conceived via a mental construction that proves it to be true." (Iemhoff 2024)*

Outline

Axiomatic set theories as a fundational system for mathematics

- *"Our axioms provide a sufficient collection of assumptions for the development of the whole of mathematics—a remarkable fact." (Enderton 1977, p. 11)*
- *"But why axiomatize set theory in the first place? Well, for one thing, it is well known that set theory provides a unified framework for the whole of pure mathematics, and surely if anything deserves to be put on a sound basis it is such a foundational subject." (Devlin [1979] 1993, p. 29)*
- *"Conventional mathematics is based on* ZFC *(the Zermelo–Fraenkel axioms, including the Axiom of Choice). Working withing* ZFC*, on develops:...All the mathematics found in basic texts on analysis, topology, algebra, etc." (Kunen [2011] 2013, p. 1)*

Axiomatic Set Theories

Some axiomatic systems of set theory

- Zermelo–Fraenkel set theory (ZF)
- Zermelo–Fraenkel set theory with Choice (ZFC)
- von Neumann–Bernays–Gödel set theory (NBG)
- Morse–Kelley set theory (MK)
- Tarski–Grothendieck set theory (TG)

Axiomatic Set Theories

But!

> *"On the other hand, we do not claim that every true fact about sets can be derived from the axioms we present. The axiomatic system is not complete in this sense." (Hrbacek and Jech [1978] 1999, p. 3)*

Foundations of Mathematics

Foundational systems

(i) Set theories (with additional axioms)
(ii) Category theories
(iii) Type theories
(iv) Univalent foundations
(v) Homotopy type theories

Foundations of Mathematics

Foundational systems[†]

(i) Set theories (with additional axioms)
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(iii) Type theories
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(v) Homotopy type theories

[†]See, e.g. (Centrone, Kant and Sarikaya 2019).

Outline

References

Georg Cantor (1895). Beiträge zur Begründung der transfiniten Mengenlehre. Mathematische Annalen 46.4, pp. 481–512. DOI: 10.1007/BF02124929 (cit. on pp. 15, 28).

— [1895] (1915). Contributions to the Founding of the Theory of Transfinite Numbers. Translation of Cantor (1895). Translated, and provided with an introduction and notes, by Philip E. B. Jourdain. Dover Publications (cit. on p. 15).

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Rosalie Iemhoff (2024). Intuitionism in the Philosophy of Mathematics. In: The Stanford Encyclopedia of Philosophy. Ed. by Edward N. Zalta and Uri Nodelman. Summer 2024. Metaphysics Research Lab, Stanford University. URL: https://plato.stanford.edu/archives/sum2024/entries/intuitionism (cit. on p. 20).

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