CM0832 - MT5001 Elements of Set Theory
The Theory of Zermelo-Fraenkel with Choice

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Outline

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Preliminaries

Textbook

Karel Hrbacek and Thomas Jech ([1978] 1999). Introduction to Set Theory.

Convention

The numbers and page numbers assigned to chapters, examples, exercises, figures, quotes, sections and theorems on these slides correspond to the numbers assigned in the textbook.

Acronyms

ZFC Zermelo-Fraenkel set theory with Choice
FOL First-Order Logic

Outline

First-Order Logic

Theorem

The identity (equality) relation is an equivalence relation.

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The identity (equality) relation is an equivalence relation.
That is, for all X, Y, Z,

(a) $X \doteq X$ (reflexivity),

(b) if $X \doteq Y$, then $Y \doteq X$ (symmetry),

(c) if $X \doteq Y$ and $Y \doteq Z$, then $X \doteq Z$ (transitivity).

First-Order Logic

Definition

A **proposition** (or **statement**) is a sentence that can be assigned a truth value of true or false.

First-Order Logic

Definitions

A **propositional function** (or **property**) is a sentence containing one or more variables (parameters) that becomes a proposition when specific values are assigned to its variables.

First-Order Logic

Definitions

A **propositional function** (or **property**) is a sentence containing one or more variables (parameters) that becomes a proposition when specific values are assigned to its variables.

The **domain** of a propositional function is the domain of the discourse.

The **codomain** of a propositional function is $\{\mathrm{true}, \mathrm{false}\}$.

First-Order Logic

Theorem

The identity (equality) relation is substitutive.

First-Order Logic

Theorem

The identity (equality) relation is substitutive.
That is, let $\mathbf{P}(X)$ be a propositional function. For all X, Y, if $X \doteq Y$ and $\mathbf{P}(X)$, then $\mathbf{P}(Y)$.

Outline

Pure Sets

Definition

A **pure set** is a set such that its members are also sets.

Pure Sets

Definition

A **pure set** is a set such that its members are also sets.

Description

The ZFC theory is a first-order theory of pure sets.

First-Order Language of ZFC

Description

In our informal but axiomatic presentation of ZFC we have two primitive (undefined) concepts: set and membership relation.

First-Order Language of ZFC

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In our informal but axiomatic presentation of ZFC we have two primitive (undefined) concepts: set and membership relation.

The (binary) membership relation is denoted $\in$.

First-Order Language of ZFC

Notation

$$X \dot{\neq} Y \stackrel{\text{def}}{=} \neg(X \doteq Y),$$
$$X \dot{\notin} Y \stackrel{\text{def}}{=} \neg(X \in Y).$$

Outline

References

Karel Hrbacek and Thomas Jech [1978] (1999). Introduction to Set Theory. Third Edition, Revised and Expanded. Marcel Dekker (cit. on p. 4).