CM0832 - MT5001 Elements of Set Theory
List of Axioms

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Preliminaries

Textbook

Karel Hrbacek and Thomas Jech ([1978] 1999). Introduction to Set Theory.

Convention

The numbers and page numbers assigned to chapters, examples, exercises, figures, quotes, sections and theorems on these slides correspond to the numbers assigned in the textbook.

List of Axioms

Axioms stating the existence of sets

The Axiom of Existence. There exists a set which has no elements.

$(\exists B)(\forall x)(x \notin B).$

List of Axioms

Axioms determining properties of sets

The Axiom of Extensionality. If every element of X is an element of Y and every element of Y is an element of X, then $X \doteq Y$.

$$(\forall X)(\forall Y)[\,(\forall z)(z \in X \leftrightarrow z \in Y) \rightarrow X \doteq Y\,].$$

List of Axioms

Axioms for building sets from other sets

The Axiom Schema of Comprehension. Let $\mathbf{P}(x)$ be a unary property of x. For any set A, there is a set B such that $x \in B$, if and only if, $x \in A$ and $\mathbf{P}(x)$.

$$(\forall A)(\exists B)(\forall x)(x \in B \leftrightarrow x \in A \land \mathbf{P}(x)).$$

The Axiom of Pair. For any A and B, there is a set C such that $x \in C$, if and only if, $x \doteq A$ or $x \doteq B$.

$$(\forall A)(\forall B)(\exists C)(\forall x)(x \in C \leftrightarrow x \doteq A \lor x \doteq B).$$

The Axiom of Union. For any set S, there exists a set U such that $x \in U$, if and only if, $x \in A$ for some $A \in S$.

$$(\forall S)(\exists U)(\forall x)[\, x \in U \leftrightarrow (\exists A)(x \in A \land A \in S)\,].$$

(continued on next slide)

List of Axioms

Axioms for building sets from other sets (continuation)

The Axiom of Power Set. For any set S, there exists a set P such that $X \in P$, if and only if, $X \subseteq S$.

$$(\forall S)(\exists P)(\forall X)[\, X \in P \leftrightarrow X \subseteq S \,].$$

References

Karel Hrbacek and Thomas Jech [1978] (1999). Introduction to Set Theory. Third Edition, Revised and Expanded. Marcel Dekker (cit. on p. 2).