

CM0832 - MT5001 Elements of Set Theory
2.1 Ordered Pairs

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Outline

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Preliminaries

Textbook

Karel Hrbacek and Thomas Jech ([1978] 1999). Introduction to Set Theory.

Convention

The numbers and page numbers assigned to chapters, examples, exercises, figures, quotes, sections and theorems on these slides correspond to the numbers assigned in the textbook.

Introduction

Starting...

*“We begin our program of developing set theory as a **foundation for mathematics** by showing how various general mathematical concepts, such as relations, functions, and orderings **can be represented by sets.**” (p. 17)*

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Definition [2.]1.1

Let a and b be sets. We define the **ordered pair** of a and b , denoted (a, b) , where a is the **first coordinate** and b is the **second coordinate**, using Kuratowski (1921)'s definition, that is,

$$(a, b) \stackrel{\text{def}}{=} \{\{a\}, \{a, b\}\}.$$

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Question

Is (a, b) a set?

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Example

We show that $(\emptyset, \{\emptyset\}) \neq (\{\emptyset\}, \emptyset)$.

$$(\emptyset, \{\emptyset\}) \stackrel{\text{def}}{=} \{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} \neq \{\{\{\emptyset\}\}, \{\{\emptyset\}, \emptyset\}\} \stackrel{\text{def}}{=} (\{\emptyset\}, \emptyset).$$

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Example

Let a be a set. Then

$$\begin{aligned}(a, a) &\stackrel{\text{def}}{=} \{\{a\}, \{a, a\}\} \\ &\doteq \{\{a\}, \{a\}\} \\ &\doteq \{\{a\}\}.\end{aligned}$$

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Exercise [2.]1.1

Let a and b sets.

- (a) Prove that $(a, b) \in \mathcal{P}(\mathcal{P}(\{a, b\}))$.
- (b) Prove that $a, b \in \bigcup(a, b)$.
- (c) Prove that if $a \in A$ and $b \in A$, then $(a, b) \in \mathcal{P}(\mathcal{P}(A))$.

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Theorem [2.]1.2

Let a, a', b, b' be sets. Then,

$$(a, b) \doteq (a', b') \quad \text{if and only if} \quad a \doteq a' \quad \text{and} \quad b \doteq b'.$$

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Theorem [2.]1.2

Let a, a', b, b' be sets. Then,

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Exercise

To give a different definition of ordered pair. Note that your definition must be a set and it must satisfy Theorem [2.]1.2.

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Definitions (p. 18)

Let a , b , c and d sets. Then

$$(a, b, c) \stackrel{\text{def}}{=} ((a, b), c)$$

(ordered triples)

$$(a, b, c, d) \stackrel{\text{def}}{=} ((a, b, c), d)$$

(ordered quadruples)

$$(a) \stackrel{\text{def}}{=} a$$

(one-tuples)

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References

- Karel Hrbacek and Thomas Jech [1978] (1999). Introduction to Set Theory. Third Edition, Revised and Expanded. Marcel Dekker (cit. on p. 4).
- Casimir Kuratowski (1921). Sur la notion de l'ordre dans la Théorie des Ensembles. *Fundamenta Mathematicae* 2, pp. 161–171. DOI: [10.4064/fm-2-1-161-171](https://doi.org/10.4064/fm-2-1-161-171) (cit. on pp. 7, 8).