CM0832 - MT5001 Elements of Set Theory
2.4 Equivalences and Partions

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Semester 2026-1

Outline

Preliminaries

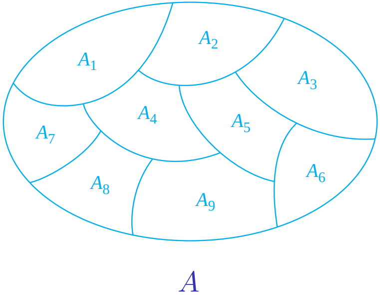
Textbook

Karel Hrbacek and Thomas Jech ([1978] 1999). Introduction to Set Theory.

Convention

The numbers and page numbers assigned to chapters, examples, exercises, figures, quotes, sections and theorems on these slides correspond to the numbers assigned in the textbook.

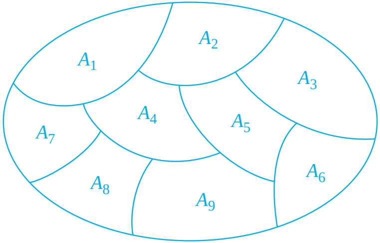
Introduction



(a) $A_i \cap A_j = \emptyset$ for $i \neq j$,

(b) $A = \bigcup A_i$.

Introduction†



A

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(b) $A = \bigcup A_i$.

†Figure source: (Rosen [1988] 2012, Fig. § 9.5, Fig. 1).

Outline

Equivalences

Definition [2.]4.1

Let R be a binary relation in a set A. The relation R is

(a) **reflexive in** A if for all $x \in A$, xRx;

(b) **symmetric in** A if for all $x, y \in A$, xRy implies yRx.

(c) **transitive in** A if for all $x, y, z \in A$, xRy and yRz imply xRz;

(d) an **equivalence on** A if R is reflexive, symmetric and transitive in A.

Equivalences

Questions

(a) Let $A = \{a, e, i, o, u\}$. Is the identity relation in A an equivalence on A?

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(a) Let $A = \{a, e, i, o, u\}$. Is the identity relation in A an equivalence on A?

(b) Let $A \not\doteq \emptyset$ be a set. Is the relation $\emptyset$ an equivalence on A? What about if $A \doteq \emptyset$?

(c) Let $A \not\doteq \emptyset$ be a set. Is the relation $A \times A$ an equivalence on A? What about if $A \doteq \emptyset$?

Equivalences

Questions

(a) Let $A = \{a, e, i, o, u\}$. Is the identity relation in A an equivalence on A?

(b) Let $A \dot{\neq} \emptyset$ be a set. Is the relation $\emptyset$ an equivalence on A? What about if $A \doteq \emptyset$?

(c) Let $A \dot{\neq} \emptyset$ be a set. Is the relation $A \times A$ an equivalence on A? What about if $A \doteq \emptyset$?

(d) Let A be a singleton. It is possible to define an equivalence on A?

Outline

Classes of Equivalence

Definition [2.]4.3

Let E be an equivalence on A and let $a \in A$. The **equivalence class of a modulo E**, denoted $[a]_E$, is the set defined by

$$[a]_E \stackrel{\text{def}}{=} \{\, x \in A \mid xEa \,\}.$$

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Partitions

Definition [2.]4.5

A system S of non-empty sets is called a partition of A if

(a) S is a system of mutually disjoint sets, i.e., if $C, D \in S$ and $C \not\doteq D$, then $C \cap D \doteq \emptyset$,

(b) the union of S is the whole set A, i.e., $\bigcup S \doteq A$.

Partitions

Definition [2.]4.8

Let S be a partition of A. The relation E_S in A is defined by

$$E_S = \{ (a, b) \in A \times A \mid \text{ there is } C \in S \text{ such that } a \in C \text{ and } b \in C \}.$$

Partitions

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Remark

Note that objects a and b are related by E_S if and only if they belong to the same set from the partition S.

Outline

Equivalences and Partitions

Theorem [2.]4.9

Let S be a partition of A; then the relation E_S is an equivalence on A.

Outline

References

Karel Hrbacek and Thomas Jech [1978] (1999). Introduction to Set Theory. Third Edition, Revised and Expanded. Marcel Dekker (cit. on p. 3).

Kenneth H. Rosen [1988] (2012). Discrete Mathematics and Its Applications. 7th ed. McGraw-Hill (cit. on p. 5).