CM0832 - MT5001 Elements of Set Theory
1.2 Properties

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Preliminaries

Textbook

Karel Hrbacek and Thomas Jech ([1978] 1999). Introduction to Set Theory.

Convention

The numbers and page numbers assigned to chapters, examples, exercises, figures, quotes, sections and theorems on these slides correspond to the numbers assigned in the textbook.

Properties

Example

We define the following unary property:

$\mathbf{P}(X)$: "There exists no $Y \in X$".

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$\mathbf{P}(X)$: "There exists no $Y \in X$".

Since that we can prove that there is a unique set X with the property $\mathbf{P}(X)$ we can add a new constant symbol $\emptyset$ denoting the empty set.

Properties

Example

We define the following binary property:

$\mathbf{P}(X, Y)$: "Every element of X is an element of Y".

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We define the following binary property:

$\mathbf{P}(X, Y)$: "Every element of X is an element of Y".

Using the above property we can add a new binary relation symbol $\subseteq$ denoting that X is a subset of Y:

$$X \subseteq Y \stackrel{\text{def}}{=} \mathbf{P}(X, Y).$$

Properties

Example

We define the following ternary property:

$\mathbf{Q}(X, Y, Z)$: “For every U, $U \in Z$ if and only if $U \in X$ and $U \in Y$”.

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$\mathbf{Q}(X, Y, Z)$: "For every U, $U \in Z$ if and only if $U \in X$ and $U \in Y$".

Since that we can prove that for every X and Y there is a unique Z such that $\mathbf{Q}(X, Y, Z)$, we can add a new binary function symbol $\cap$, denoting the intersection of X and Y:

$$X \cap Y \stackrel{\text{def}}{=} \text{the unique } Z \text{ such that } \mathbf{Q}(X, Y, Z).$$

Outline

References

Karel Hrbacek and Thomas Jech [1978] (1999). Introduction to Set Theory. Third Edition, Revised and Expanded. Marcel Dekker (cit. on p. 2).