CM0832 - MT5001 Elements of Set Theory
1.4 Elementary Operations on Sets

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Preliminaries

Textbook

Karel Hrbacek and Thomas Jech ([1978] 1999). Introduction to Set Theory.

Convention

The numbers and page numbers assigned to chapters, examples, exercises, figures, quotes, sections and theorems on these slides correspond to the numbers assigned in the textbook.

Elementary Operations on Sets

Definitions

$$A \subset B \stackrel{\text{def}}{=} A \subseteq B \text{ and } A \doteqdot\!\!\!\!/\; B$$ **(proper subset)**

$$A \supseteq B \stackrel{\text{def}}{=} A \subseteq B$$ **(superset)**

$$A \supset B \stackrel{\text{def}}{=} A \subset B$$ **(proper superset)**

Elementary Operations on Sets

Definitions

Let A and B sets. Then

$$A \cup B \stackrel{\text{def}}{=} \bigcup \{A, B\} \qquad \textbf{(union)}$$

$$A \cap B \stackrel{\text{def}}{=} \{\, x \in A \mid x \in B \,\} \qquad \textbf{(intersection)}$$

$$A - B \stackrel{\text{def}}{=} \{\, x \in A \mid x \notin B \,\} \qquad \textbf{(difference)}$$

$$A \bigtriangleup B \stackrel{\text{def}}{=} (A - B) \cup (B - A) \qquad \textbf{(symmetric difference)}$$

Elementary Operations on Sets

Convention (pp. 9–10)

*“We say that S is a **system of sets** or a **collection of sets** when we want to stress that elements of S are sets (of course, this is always true—all our objects are sets—and thus the expressions ‘set’ and ‘system of sets’ have the same meaning).”*

Elementary Operations on Sets

Exercise

To prove that for any non-empty system of sets S, there exists a unique set B such that $x \in B$ if and only if x belongs to every member of S.

Elementary Operations on Sets

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To prove that for any non-empty system of sets S, there exists a unique set B such that $x \in B$ if and only if x belongs to every member of S.

Proof

(a) We prove the existence of the set B from the hypothesis that exists $A \in S$ and the Axiom Schema of Comprehension on property $\mathbf{P}(x, S)$: "x belongs to every member of S".

(b) We prove that the set B is unique from the Axiom of Extensionality.

Elementary Operations on Sets

Exercise

To prove that for any non-empty system of sets S, there exists a unique set B such that $x \in B$ if and only if x belongs to every member of S.

Definition (p. 14)

The (**generalised**) **intersection** of a non-empty system of sets S, denoted $\bigcap S$, is the set postulated by the exercise.

Elementary Operations on Sets

Question

Note that $\bigcap \emptyset$ is not defined. Why?

Elementary Operations on Sets

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Alternative definition

$A \cap B \stackrel{\text{def}}{=} \bigcap \{A, B\}$.

Elementary Operations on Sets

Definitions (p. 14)

Sets A and B are **disjoint** if $A \cap B \doteq \emptyset$.

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Sets A and B are **disjoint** if $A \cap B \doteq \emptyset$.

A system of sets S is **mutually disjoint** (or **pairwise disjoint**) if $A \cap B \doteq \emptyset$, for all $A, B \in S$ with $A \not\doteq B$.

Outline

References

References

Karel Hrbacek and Thomas Jech [1978] (1999). Introduction to Set Theory. Third Edition, Revised and Expanded. Marcel Dekker (cit. on p. 2).