CM0832 Elements of Set Theory
List of Axioms

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Preliminaries

Textbook

Enderton (1977). Elements of Set Theory.

Convention

The numbers and page numbers assigned to chapters, examples, exercises, figures, quotes, sections and theorems on these slides correspond to the numbers assigned in the textbook.

List of Axioms

Axioms stating the existence of sets

- **Empty (existence) axiom**: There is a set having no members, that is,

$$\exists B \, \forall x \, (x \notin B).$$

- **Infinity axiom**: There exists an inductive set, that is,

$$\exists A \, [\, \emptyset \in A \wedge \forall a \, (a \in A \rightarrow a^+ \in A) \,].$$

List of Axioms

Axioms determining properties of sets

- **Extensionality axiom**: If two sets have exactly the same members, then they are equal, that is,

$$\forall A\,\forall B\,[\,\forall x\,(x \in A \leftrightarrow x \in B) \to A = B\,].$$

- **Regularity (foundation) axiom**: All sets are well-founded, that is,

$$\forall A\,[\,A \neq \emptyset \to \exists m\,(m \in A \wedge m \cap A = \emptyset)\,].$$

List of Axioms

Axioms for building sets from other sets

- **Pairing axiom**: For any sets u and v, there is a set having as members just u and v, that is,

$$\forall a\,\forall b\,\exists C\,\forall x\,(x \in C \leftrightarrow x = a \vee x = b).$$

- **Union axiom** (first version): For any sets a and b, there is a set whose members are those sets belonging either to a or to b (or both), that is,

$$\forall a\,\forall b\,\exists B\,\forall x\,(x \in B \leftrightarrow x \in a \vee x \in b).$$

- **Union axiom** (final version): For any set A, there exists a set B whose elements are exactly the members of the members of A, that is,

$$\forall A\,\exists B\,\forall x\,[\,x \in B \leftrightarrow \exists b\,(x \in b \wedge b \in A)\,].$$

List of Axioms

Axioms for building sets from other sets (continuation)

- **Power set axiom**: For any set a, there is a set whose members are exactly the subsets of a, that is,

$$\forall a \, \exists B \, \forall x \, (x \in B \leftrightarrow x \subseteq a).$$

- **Subset axiom scheme** (axiom scheme of comprehension or separation): For any propositional function $\varphi(x, t_1, \ldots, t_k)$, not containing B, the following is an axiom:

$$\forall t_1 \cdots \forall t_k \, \forall c \, \exists B \, \forall x \, (x \in B \leftrightarrow x \in c \wedge \varphi(x, t_1, \ldots, t_k)).$$

- **Axiom of choice** (a version): For any relation R there is a function $F \subseteq R$ with $\operatorname{dom} F = \operatorname{dom} R$.

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List of Axioms

Axioms for building sets from other sets (continuation)

- **Replacement axiom scheme**: For any propositional function $\varphi(x, y, t_1, \ldots, t_k)$, not containing B, the following is an axiom:

$$\forall t_1 \cdots \forall t_k \, \forall A \, [\, \forall x \, (x \in A \to \exists! y \, \varphi(x, y, t_1, \ldots, t_k)) \to \\ \exists B \, \forall y \, (y \in B \leftrightarrow \exists x \, (x \in A \land \varphi(x, y, t_1, \ldots, t_k))) \,].$$

References

Herbert B. Enderton (1977). Elements of Set Theory. Academic Press (cit. on p. 2).